6.2 Η ΕΝΝΟΙΑ ΤΗΣ ΑΚΟΛΟΥΘΙΑΣ

Δραστηριότητες σελ. 86 (Η έννοια της ακολουθίας)

1. (α) $\alpha_\nu = \nu + 2,\ \nu \in \mathbb{N}$

$\nu = 1 \quad \rightsquigarrow \quad \alpha_1 = 1 + 2 = 3$

$\nu = 2 \quad \rightsquigarrow \quad \alpha_2 = 2 + 2 = 4$

$\nu = 3 \quad \rightsquigarrow \quad \alpha_3 = 3 + 2 = 5$

$\nu = 4 \quad \rightsquigarrow \quad \alpha_4 = 4 + 2 = 6$

$\nu = 5 \quad \rightsquigarrow \quad \alpha_5 = 5 + 2 = 7$

$\nu = 6 \quad \rightsquigarrow \quad \alpha_6 = 6 + 2 = 8$

(β) $\alpha_\nu = \nu^2 - 3,\ \nu \in \mathbb{N}$

$\nu = 1 \quad \rightsquigarrow \quad \alpha_1 = 1^2 - 3 = -2$

$\nu = 2 \quad \rightsquigarrow \quad \alpha_2 = 2^2 - 3 = 1$

$\nu = 3 \quad \rightsquigarrow \quad \alpha_3 = 3^2 - 3 = 6$

$\nu = 4 \quad \rightsquigarrow \quad \alpha_4 = 4^2 - 3 = 13$

$\nu = 5 \quad \rightsquigarrow \quad \alpha_5 = 5^2 - 3 = 22$

$\nu = 6 \quad \rightsquigarrow \quad \alpha_6 = 6^2 - 3 = 33$

(γ) $\alpha_\nu = 3^\nu,\ \nu \in \mathbb{N}$

$\nu = 1 \quad \rightsquigarrow \quad \alpha_1 = 3^1 = 3$

$\nu = 2 \quad \rightsquigarrow \quad \alpha_2 = 3^2 = 9$

$\nu = 3 \quad \rightsquigarrow \quad \alpha_3 = 3^3 = 27$

$\nu = 4 \quad \rightsquigarrow \quad \alpha_4 = 3^4 = 81$

$\nu = 5 \quad \rightsquigarrow \quad \alpha_5 = 3^5 = 243$

$\nu = 6 \quad \rightsquigarrow \quad \alpha_6 = 3^6 = 729$

(δ) $\alpha_\nu = (-4)^\nu,\ \nu \in \mathbb{N}$

$\nu = 1 \quad \rightsquigarrow \quad \alpha_1 = (-4)^1 = -4$

$\nu = 2 \quad \rightsquigarrow \quad \alpha_2 = (-4)^2 = 16$

$\nu = 3 \quad \rightsquigarrow \quad \alpha_3 = (-4)^3 = -64$

$\nu = 4 \quad \rightsquigarrow \quad \alpha_4 = (-4)^4 = 256$

$\nu = 5 \quad \rightsquigarrow \quad \alpha_5 = (-4)^5 = -1024$

$\nu = 6 \quad \rightsquigarrow \quad \alpha_6 = (-4)^6 = 4096$

(ε) $\alpha_\nu = (-1)^\nu + 1,\ \nu \in \mathbb{N}$

$\nu = 1 \quad \rightsquigarrow \quad \alpha_1 = (-1)^1 + 1 = -1 + 1 = 0$

$\nu = 2 \quad \rightsquigarrow \quad \alpha_2 = (-1)^2 + 1 = 1 + 1 = 2$

$\nu = 3 \quad \rightsquigarrow \quad \alpha_3 = (-1)^3 + 1 = -1 + 1 = 0$

$\nu = 4 \quad \rightsquigarrow \quad \alpha_4 = (-1)^4 + 1 = 1 + 1 = 2$

$\nu = 5 \quad \rightsquigarrow \quad \alpha_5 = (-1)^5 + 1 = -1 + 1 = 0$

$\nu = 6 \quad \rightsquigarrow \quad \alpha_6 = (-1)^6 + 1 = 1 + 1 = 2$

(στ) $\alpha_\nu = \eta\mu\left(\frac{\nu\pi}{2}\right),\ \nu \in \mathbb{N}$

$\nu = 1 \quad \rightsquigarrow \quad \alpha_1 = \eta\mu\left(\frac{\pi}{2}\right) = 1$

$\nu = 2 \quad \rightsquigarrow \quad \alpha_2 = \eta\mu\left(\frac{2\pi}{2}\right) = \eta\mu(\pi) = 0$

$\nu = 3 \quad \rightsquigarrow \quad \alpha_3 = \eta\mu\left(\frac{3\pi}{2}\right) = -1$

$\nu = 4 \quad \rightsquigarrow \quad \alpha_4 = \eta\mu\left(\frac{4\pi}{2}\right) = \eta\mu(2\pi) = 0$

$\nu = 5 \quad \rightsquigarrow \quad \alpha_5 = \eta\mu\left(\frac{5\pi}{2}\right) = 1$

$\nu = 6 \quad \rightsquigarrow \quad \alpha_6 = \eta\mu\left(\frac{6\pi}{2}\right) = \eta\mu(3\pi) = 0$

(ζ) $\alpha_{\nu+1} = \alpha_\nu - 2,\ \alpha_1 = 2$

$\nu = 1 \Rightarrow \alpha_{1+1} = \alpha_1 - 2 \Rightarrow \alpha_2 = 2 - 2 = 0$
$\nu = 2 \Rightarrow \alpha_{2+1} = \alpha_2 - 2 \Rightarrow \alpha_3 = 0 - 2 = -2$
$\nu = 3 \Rightarrow \alpha_{3+1} = \alpha_3 - 2 \Rightarrow \alpha_4 = -2 - 2 = -4$
$\nu = 4 \Rightarrow \alpha_{4+1} = \alpha_4 - 2 \Rightarrow \alpha_5 = -4 - 2 = -6$
$\nu = 5 \Rightarrow \alpha_{5+1} = \alpha_5 - 2 \Rightarrow \alpha_6 = -6 - 2 = -8$

(η) $\frac{\alpha_{\nu+1}}{\alpha_\nu} = 4,\ \alpha_1 = -3$, δηλ.

$$\alpha_{\nu+1} = 4\alpha_\nu, \qquad \alpha_1 = -3$$

$\nu = 1 \Rightarrow \alpha_{1+1} = 4\alpha_1 \Rightarrow \alpha_2 = -12$
$\nu = 2 \Rightarrow \alpha_{2+1} = 4\alpha_2 \Rightarrow \alpha_3 = -48$
$\nu = 3 \Rightarrow \alpha_{3+1} = 4\alpha_3 \Rightarrow \alpha_4 = -192$
$\nu = 4 \Rightarrow \alpha_{4+1} = 4\alpha_4 \Rightarrow \alpha_5 = -768$
$\nu = 5 \Rightarrow \alpha_{5+1} = 4\alpha_5 \Rightarrow \alpha_6 = -3072$

(θ) $\alpha_{\nu+1} = {\alpha_\nu}^3,\ \alpha_1 = -1$

$\nu = 1 \Rightarrow \alpha_{1+1} = {\alpha_1}^3 = (-1)^3 \Rightarrow \alpha_2 = -1$
$\nu = 2 \Rightarrow \alpha_{2+1} = {\alpha_2}^3 = (-1)^3 \Rightarrow \alpha_3 = -1$
$\nu = 3 \Rightarrow \alpha_{3+1} = {\alpha_3}^3 = (-1)^3 \Rightarrow \alpha_4 = -1$
$\nu = 4 \Rightarrow \alpha_{4+1} = {\alpha_4}^3 = (-1)^3 \Rightarrow \alpha_5 = -1$
$\nu = 5 \Rightarrow \alpha_{5+1} = {\alpha_5}^3 = (-1)^3 \Rightarrow \alpha_6 = -1$

2. ❷ **(α)** $\alpha_\nu = 3\nu - 2,\ \nu \in \mathbb{N}$

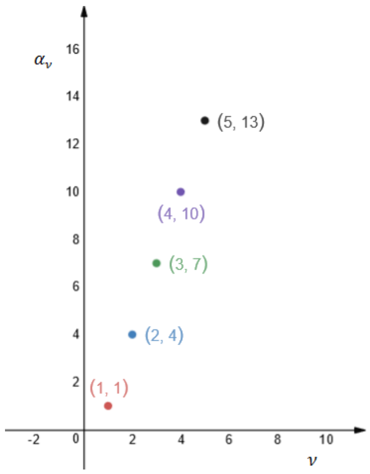
$\nu = 1 \quad \rightsquigarrow \quad \alpha_1 = 3 \cdot 1 - 2 = 1$

$\nu = 2 \quad \rightsquigarrow \quad \alpha_2 = 3 \cdot 2 - 2 = 4$

$\nu = 3 \quad \rightsquigarrow \quad \alpha_3 = 3 \cdot 3 - 2 = 7$

$\nu = 4 \quad \rightsquigarrow \quad \alpha_4 = 3 \cdot 4 - 2 = 10$

$\nu = 5 \quad \rightsquigarrow \quad \alpha_5 = 3 \cdot 5 - 2 = 13$



(β) $\alpha_\nu = \frac{(-1)^\nu}{\nu},\ \nu \in \mathbb{N}$

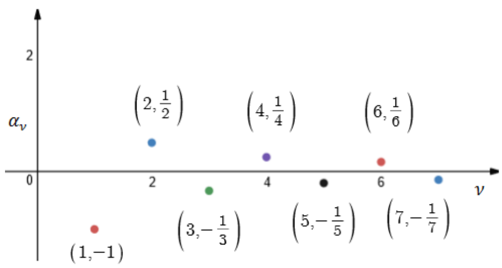
$\nu = 1 \quad \rightsquigarrow \quad \alpha_1 = -1$

$\nu = 2 \quad \rightsquigarrow \quad \alpha_2 = \frac{1}{2}$

$\nu = 3 \quad \rightsquigarrow \quad \alpha_3 = -\frac{1}{3}$

$\nu = 4 \quad \rightsquigarrow \quad \alpha_4 = \frac{1}{4}$

$\nu = 5 \quad \rightsquigarrow \quad \alpha_5 = 3 \cdot 5 - 2 = 13$



(γ) $\alpha_\nu = \eta\mu\left(\frac{\nu\pi}{4}\right),\ \nu \in \mathbb{N}$

$\nu = 1 \quad \rightsquigarrow \quad \alpha_1 = \eta\mu\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

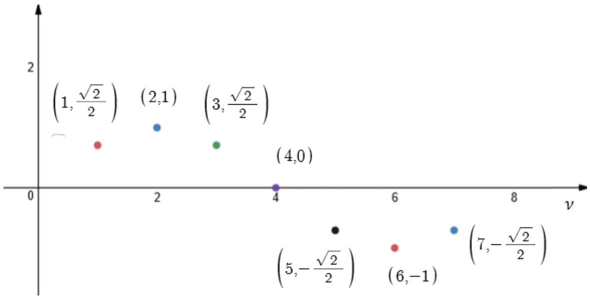
$\nu = 2 \quad \rightsquigarrow \quad \alpha_2 = \eta\mu\left(\frac{2\pi}{4}\right) = \eta\mu\left(\frac{\pi}{2}\right) = 1$

$\nu = 3 \quad \rightsquigarrow \quad \alpha_3 = \eta\mu\left(\frac{3\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$\nu = 4 \quad \rightsquigarrow \quad \alpha_4 = \eta\mu\left(\frac{4\pi}{4}\right) = \eta\mu(\pi) = 0$

$\nu = 5 \quad \rightsquigarrow \quad \alpha_5 = \eta\mu\left(\frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

$\nu = 6 \quad \rightsquigarrow \quad \alpha_6 = \eta\mu\left(\frac{6\pi}{4}\right) = \eta\mu\left(\frac{3\pi}{2}\right) = -1$



3.

$\alpha_{\nu+1} = 3\alpha_\nu - 4, \qquad \alpha_1 = 2$

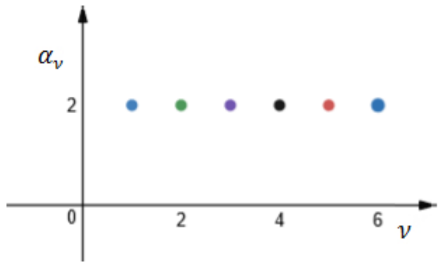
$\nu = 1 \Rightarrow \alpha_{1+1} = 3\alpha_1 - 4 \Rightarrow \alpha_2 = 2$

$\nu = 2 \Rightarrow \alpha_{2+1} = 3\alpha_2 - 4 \Rightarrow \alpha_3 = 2$

$\nu = 3 \Rightarrow \alpha_{3+1} = 3\alpha_3 - 4 \Rightarrow \alpha_4 = 2$

$\nu = 4 \Rightarrow \alpha_{4+1} = 3\alpha_4 - 4 \Rightarrow \alpha_5 = 2$

$\nu = 5 \Rightarrow \alpha_{5+1} = 3\alpha_5 - 4 \Rightarrow \alpha_6 = 2$



4.

$$\alpha_\nu = \nu^2 + \nu + 1, \qquad \nu \in \mathbb{N}$$

Έχουμε

$\alpha_\nu = 57 \Leftrightarrow \nu^2 + \nu + 1 = 57 \Leftrightarrow \nu^2 + \nu - 56 = 0$
$\Leftrightarrow (\nu - 7)(\nu + 8) = 0 \Leftrightarrow \boldsymbol{\nu = 7, -8}$.
Η τιμή $\nu = -8$ απορρίπτεται διότι το ν είναι φυσικός αριθμός (άρα θετικός).

5. **(α)**

$$-1, 2, -3, 4, \cdots$$

Παρατηρούμε ότι

$$\alpha_1 = -1 = (-1)^1 \cdot 1$$
$$\alpha_2 = 2 = (-1)^2 \cdot 2$$
$$\alpha_3 = -3 = (-1)^3 \cdot 3$$
$$\alpha_4 = 4 = (-1)^4 \cdot 4$$
$$\vdots$$

και άρα

$$\boldsymbol{\alpha_\nu = (-1)^\nu \cdot \nu \quad (\nu \in \mathbb{N})}$$

(β) Παρατηρούμε ότι

$$\alpha_1 = \frac{3}{4} = \frac{1+2}{1+3}$$
$$\alpha_2 = \frac{4}{5} = \frac{2+2}{2+3}$$
$$\alpha_3 = \frac{5}{6} = \frac{3+2}{3+3}$$

	$$\alpha_4 = \frac{6}{7} = \frac{4+2}{4+3}$$ και άρα $$\boldsymbol{\alpha_\nu = \frac{\nu+2}{\nu+3}}, \qquad (\boldsymbol{\nu} \in \mathbb{N})$$
6.	$$\alpha_\nu = 1+3+5+\cdots+(2\nu-1), \qquad \nu \in \mathbb{N}$$ **(α)** Έχουμε $\alpha_1 = 1, \alpha_2 = 1+3 = 4, \alpha_3 = 1+3+5 = 9$ και για $\nu = 4$, έχουμε $$\alpha_4 = 1+3+5+(2\cdot 4-1) = 16$$ **(β)** Για $\nu \in \mathbb{N}$ έχουμε $$\begin{aligned}\alpha_{\nu+1} - \alpha_\nu &= \big(1+3+5+\cdots+(2\nu-1)+(2(\nu+1)-1)\big) - \big(1+3+5+\cdots+(2\nu-1)\big)\\ &= (2(\nu+1)-1) = 2\nu+1\end{aligned}$$ **(γ)** Από το ερώτημα (α) υποψιαζόμαστε ότι $\alpha_\nu = \nu^2$ το οποίο θα το επαληθεύσουμε με τη μέθοδο της επαγωγής: Ο προτασιακός τύπος $P(\nu)$ του οποίου πρέπει να αποδείξουμε την αλήθεια στο σύνολο των φυσικών αριθμών είναι ο $P(\nu)\colon \alpha_\nu = \nu^2$ **Βήμα 1** Για $\nu = 1$ είναι $\alpha_1 = 1 = 1^2$ και αρα ο $P(1)$ είναι αληθής. **Βήμα 2** Υποθέτουμε ότι ο $P(\kappa)$ είναι αληθής για κάποιο $\kappa \in \mathbb{N}, \kappa \geq 2$, δηλ. ότι $\alpha_\kappa = \kappa^2$. Τότε $$\alpha_{\kappa+1} = \alpha_\kappa + (2(\kappa+1)-1) = \kappa^2 + 2\kappa + 1 = (\kappa+1)^2$$ και το αποτέλεσμα ισχύει, δηλ. ο προτασιακός τύπος $P(\kappa+1)$ είναι αληθής **Βήμα 3** Από την Αρχή της Μαθηματικής επαγωγής, έπεται ότι ο προτασιακός τύπος $P(\nu)\colon \alpha_\nu = \nu^2$ έχει σύνολο αλήθειας το $\mathbb{N}$.