4.2 ΤΡΙΓΩΝΟΜΕΤΡΙΚΕΣ ΕΞΙΣΩΣΕΙΣ

Δραστηριότητες σελ. 194-195 (Τριγωνομετρικές εξισώσεις)

1.

(α) Έχουμε

$$\eta\mu x = \eta\mu\left(\frac{\pi}{5}\right) \Leftrightarrow \left(x = 2\kappa\pi + \frac{\pi}{5}, \kappa \in \mathbb{Z}\right) \vee \left(x = (2\kappa+1)\pi - \frac{\pi}{5}, \kappa \in \mathbb{Z}\right)$$
$$\Leftrightarrow \left(\boldsymbol{x = 2\kappa\pi + \frac{\pi}{5}, \kappa \in \mathbb{Z}}\right) \vee \left(\boldsymbol{x = 2\kappa\pi + \frac{4\pi}{5}, \kappa \in \mathbb{Z}}\right)$$

(β) Έχουμε

$$\eta\mu x = 1 \Leftrightarrow \eta\mu x = \eta\mu\left(\frac{\pi}{2}\right)$$
$$\Leftrightarrow \left(x = 2\kappa\pi + \frac{\pi}{2}, \kappa \in \mathbb{Z}\right) \vee \left(x = (2\kappa+1)\pi - \frac{\pi}{2}, \kappa \in \mathbb{Z}\right)$$
$$\Leftrightarrow \boldsymbol{x = 2\kappa\pi + \frac{\pi}{2}, \kappa \in \mathbb{Z}}$$

(γ) Έχουμε

$$\eta\mu(3x) = \frac{1}{2} \Leftrightarrow \eta\mu x = \eta\mu\left(\frac{\pi}{6}\right)$$
$$\Leftrightarrow \left(3x = 2\kappa\pi + \frac{\pi}{6}, \kappa \in \mathbb{Z}\right) \vee \left(x = (2\kappa+1)\pi - \frac{\pi}{6}, \kappa \in \mathbb{Z}\right)$$
$$\Leftrightarrow \left(\boldsymbol{x = \frac{2}{3}\kappa\pi + \frac{\pi}{18}, \kappa \in \mathbb{Z}}\right) \vee \left(\boldsymbol{x = \frac{2}{3}\kappa\pi + \Leftrightarrow \frac{5\pi}{18}, \kappa \in \mathbb{Z}}\right)$$

(δ) Έχουμε

$$\eta\mu\left(3x - \frac{\pi}{4}\right) = \frac{1}{2} \Leftrightarrow \eta\mu\left(3x - \frac{\pi}{4}\right) = \eta\mu\left(\frac{\pi}{6}\right)$$
$$\Leftrightarrow \left(3x - \frac{\pi}{4} = 2\kappa\pi + \frac{\pi}{6}, \kappa \in \mathbb{Z}\right) \vee \left(3x - \frac{\pi}{4} = (2\kappa+1)\pi - \frac{\pi}{6}, \kappa \in \mathbb{Z}\right)$$
$$\Leftrightarrow \left(3x = 2\kappa\pi + \frac{\pi}{6} + \frac{\pi}{4}, \kappa \in \mathbb{Z}\right) \vee \left(3x = 2\kappa\pi + \frac{5\pi}{6} + \frac{\pi}{4}, \kappa \in \mathbb{Z}\right)$$
$$\Leftrightarrow \left(3x = 2\kappa\pi + \frac{5\pi}{12}, \kappa \in \mathbb{Z}\right) \vee \left(3x = 2\kappa\pi + \frac{13\pi}{12}, \kappa \in \mathbb{Z}\right)$$
$$\Leftrightarrow \left(\boldsymbol{x = \frac{2}{3}\kappa\pi + \frac{5\pi}{36}, \kappa \in \mathbb{Z}}\right) \vee \left(\boldsymbol{x = \frac{2}{3}\kappa\pi + \frac{13\pi}{36}, \kappa \in \mathbb{Z}}\right)$$

(ε) Είναι

$$\eta\mu(2x) - 2 = 0 \Leftrightarrow \eta\mu(2x) = 2$$

η οποία είναι **αδύνατη** αφού $\eta\mu(2x) \in [-1,1]$

(στ) Έχουμε

$$2\eta\mu x + \sqrt{3} = 0 \Leftrightarrow 2\eta\mu x = -\sqrt{3} \Leftrightarrow \eta\mu x = -\frac{\sqrt{3}}{2} \Leftrightarrow \eta\mu x = \eta\mu\left(\frac{4\pi}{3}\right)$$
$$\Leftrightarrow \left(x = 2\kappa\pi + \frac{4\pi}{3}, \kappa \in \mathbb{Z}\right) \vee \left(x = (2\kappa+1)\pi - \frac{4\pi}{3}, \kappa \in \mathbb{Z}\right)$$
$$\Leftrightarrow \left(\boldsymbol{x = 2\kappa\pi + \frac{4\pi}{3}, \kappa \in \mathbb{Z}}\right) \vee \left(\boldsymbol{x = 2\kappa\pi - \frac{\pi}{3}, \kappa \in \mathbb{Z}}\right)$$

2.

(α) Έχουμε

$$\sigma\upsilon\nu x = \sigma\upsilon\nu\left(\frac{\pi}{7}\right) \Leftrightarrow \boldsymbol{x = 2\kappa\pi \pm \frac{\pi}{7}, \kappa \in \mathbb{Z}}$$

(β) Έχουμε

$$\sigma\upsilon\nu(2x) = \frac{\sqrt{2}}{2} \Leftrightarrow \sigma\upsilon\nu(2x) = \sigma\upsilon\nu\left(\frac{\pi}{4}\right)$$
$$\Leftrightarrow \left(2x = 2\kappa\pi + \frac{\pi}{4}, \kappa \in \mathbb{Z}\right) \vee \left(2x = 2\kappa\pi - \frac{\pi}{4}, \kappa \in \mathbb{Z}\right)$$
$$\Leftrightarrow \boldsymbol{x = \kappa\pi \pm \frac{\pi}{8}, \kappa \in \mathbb{Z}}$$

(γ) Έχουμε

$$2\sigma\upsilon\nu(2x)-1=0 \Leftrightarrow \sigma\upsilon\nu(2x)=\frac{1}{2} \Leftrightarrow \sigma\upsilon\nu(2x)=\frac{\pi}{3}$$
$$\Leftrightarrow \left(2x=2\kappa\pi+\frac{\pi}{3},\kappa\in\mathbb{Z}\right)\vee\left(2x=2\kappa\pi-\frac{\pi}{3},\kappa\in\mathbb{Z}\right)$$
$$\Leftrightarrow \boldsymbol{x=\kappa\pi\pm\frac{\pi}{6},\kappa\in\mathbb{Z}}$$

(δ) Είναι **αδύνατη** αφού $\sigma\upsilon\nu(4x)\in[-1,1]$

(ε) Έχουμε

$$\sigma\upsilon\nu\left(x-\frac{\pi}{4}\right)=\sigma\upsilon\nu\left(2x-\frac{\pi}{3}\right)$$
$$\Leftrightarrow \left(x-\frac{\pi}{4}=2\kappa\pi+2x-\frac{\pi}{3},\kappa\in\mathbb{Z}\right)\vee\left(2x-\frac{\pi}{3}=2\kappa\pi-\left(2x-\frac{\pi}{3}\right),\kappa\in\mathbb{Z}\right)$$
$$\Leftrightarrow \left(-x=2\kappa\pi+\frac{\pi}{4}-\frac{\pi}{3},\kappa\in\mathbb{Z}\right)\vee\left(3x=2\kappa\pi+\frac{\pi}{3}+\frac{\pi}{4},\kappa\in\mathbb{Z}\right)$$
$$\Leftrightarrow \left(\boldsymbol{x=\frac{\pi}{12}-2\kappa\pi},\boldsymbol{\kappa}\in\mathbb{Z}\right)\vee\left(\boldsymbol{x=\frac{2}{3}\kappa\pi+\frac{7\pi}{36}},\boldsymbol{\kappa}\in\mathbb{Z}\right)$$
$$\Leftrightarrow \left(\boldsymbol{x=2\kappa\pi+\frac{\pi}{12}},\boldsymbol{\kappa}\in\mathbb{Z}\right)\vee\left(\boldsymbol{x=\frac{2}{3}\kappa\pi+\frac{7\pi}{36}},\boldsymbol{\kappa}\in\mathbb{Z}\right)$$

(το αρνητικό πρόσημο απορροφήθηκε στο κ)

(στ) Έχουμε

$$\sigma\upsilon\nu\left(x-\frac{\pi}{3}\right)=\eta\mu x \Leftrightarrow \sigma\upsilon\nu\left(x-\frac{\pi}{3}\right)=\sigma\upsilon\nu\left(\frac{\pi}{2}-x\right)$$
$$\Leftrightarrow \left(x-\frac{\pi}{3}=2\kappa\pi+\frac{\pi}{2}-x,\kappa\in\mathbb{Z}\right)\vee\left(x-\frac{\pi}{3}=2\kappa\pi-\left(\frac{\pi}{2}-x\right),\kappa\in\mathbb{Z}\right)$$
$$\Leftrightarrow \left(2x=2\kappa\pi+\frac{\pi}{2}+\frac{\pi}{3},\kappa\in\mathbb{Z}\right) \Leftrightarrow \left(2x=2\kappa\pi+\frac{5\pi}{6},\kappa\in\mathbb{Z}\right)$$
$$\Leftrightarrow \boldsymbol{x=\kappa\pi+\frac{5\pi}{12}},\boldsymbol{\kappa}\in\mathbb{Z}$$

3. **(α)** Έχουμε

$$\varepsilon\varphi x=\varepsilon\varphi\left(\frac{\pi}{9}\right) \Leftrightarrow \boldsymbol{x=\kappa\pi+\frac{\pi}{9}},\boldsymbol{\kappa}\in\mathbb{Z}$$

(β) Έχουμε

$$\sigma\varphi(2x)=\sigma\varphi\left(\frac{\pi}{11}\right) \Leftrightarrow 2x=\kappa\pi+\frac{\pi}{11},\kappa\in\mathbb{Z}$$
$$\Leftrightarrow \boldsymbol{x=\frac{\kappa\pi}{2}+\frac{\pi}{22}},\boldsymbol{\kappa}\in\mathbb{Z}$$

(γ) Έχουμε

$$\varepsilon\varphi(3x)=1 \Leftrightarrow \varepsilon\varphi(3x)=\varepsilon\varphi\left(\frac{\pi}{4}\right) \Leftrightarrow 3x=\kappa\pi+\frac{\pi}{4},\kappa\in\mathbb{Z} \Leftrightarrow \boldsymbol{x=\frac{\kappa\pi}{3}+\frac{\pi}{12}},\boldsymbol{\kappa}\in\mathbb{Z}$$

(δ) Έχουμε

$$3\sigma\varphi x=-\sqrt{3} \Leftrightarrow \sigma\varphi x=-\frac{\sqrt{3}}{3} \Leftrightarrow \varepsilon\varphi x=-\sqrt{3}$$
$$\Leftrightarrow \sigma\varphi x=\sigma\varphi\left(-\frac{\pi}{3}\right) \Leftrightarrow x=\kappa\pi+\left(-\frac{\pi}{3}\right), \quad \kappa\in\mathbb{Z}$$
$$\Leftrightarrow \boldsymbol{x=\kappa\pi-\frac{\pi}{3}}, \quad \boldsymbol{\kappa}\in\mathbb{Z}$$

(ε) Έχουμε

$$\varepsilon\varphi\left(3x-\frac{\pi}{4}\right)=\varepsilon\varphi\left(x-\frac{\pi}{3}\right) \Leftrightarrow 3x-\frac{\pi}{4}=\kappa\pi+\left(x-\frac{\pi}{3}\right),\kappa\in\mathbb{Z}$$
$$\Leftrightarrow 2x=\kappa\pi+\frac{\pi}{4}-\frac{\pi}{3}, \quad \kappa\in\mathbb{Z} \Leftrightarrow \boldsymbol{x=\frac{\kappa\pi}{2}-\frac{\pi}{24}},\boldsymbol{\kappa}\in\mathbb{Z}$$

(στ) Έχουμε

$$\varepsilon\varphi x=\sigma\varphi x \Leftrightarrow \varepsilon\varphi x=\varepsilon\varphi\left(\frac{\pi}{2}-x\right), \quad \kappa\in\mathbb{Z}$$
$$\Leftrightarrow x=\kappa\pi+\frac{\pi}{2}-x, \ \kappa\in\mathbb{Z} \Leftrightarrow 2x=\kappa\pi+\frac{\pi}{2}, \quad \kappa\in\mathbb{Z}$$

$$\Leftrightarrow \boldsymbol{x = \frac{\kappa\pi}{2} + \frac{\pi}{4}}, \qquad \boldsymbol{\kappa \in \mathbb{Z}}$$

4. **(α)** Έχουμε

$$(\eta\mu x - 1)\cdot(2\eta\mu x + 1) = 0 \Leftrightarrow \left(\eta\mu x = \eta\mu\left(\frac{\pi}{2}\right)\right) \vee \left(\eta\mu x = \left(-\frac{\pi}{6}\right)\right)$$

$$\Leftrightarrow \left(\eta\mu x = 2\kappa\pi + \frac{\pi}{2}, \kappa \in \mathbb{Z}\right) \vee \left(\eta\mu x = \eta\mu\left(-\frac{\pi}{6}\right)\right)$$

$$\Leftrightarrow \left(\eta\mu x = 2\kappa\pi + \frac{\pi}{2}, \kappa \in \mathbb{Z}\right) \vee \left(\eta\mu x = 2\kappa\pi + \left(-\frac{\pi}{6}\right), \kappa \in \mathbb{Z}\right) \vee \left(\eta\mu x = 2\kappa\pi + \left(-\frac{\pi}{6}\right), \kappa \in \mathbb{Z}\right)$$

$$\Leftrightarrow \left(\boldsymbol{x = 2\kappa\pi + \frac{\pi}{2}, \kappa \in \mathbb{Z}}\right) \vee \left(\boldsymbol{x = 2\kappa\pi + \frac{7\pi}{6}, \kappa \in \mathbb{Z}}\right) \vee \left(\boldsymbol{x = 2\kappa\pi - \frac{\pi}{6}, \kappa \in \mathbb{Z}}\right)$$

(β) Έχουμε

$$\sigma\upsilon\nu x \cdot (1 - \sigma\upsilon\nu x) = 0 \Leftrightarrow (\sigma\upsilon\nu x = 0) \vee (\sigma\upsilon\nu x = 1) \Leftrightarrow \left(\sigma\upsilon\nu x = \sigma\upsilon\nu\left(\frac{\pi}{2}\right)\right) \vee (\sigma\upsilon\nu x = \sigma\upsilon\nu 0)$$

$$\Leftrightarrow \begin{cases} \boldsymbol{x = 2\kappa\pi \pm \frac{\pi}{2}, \kappa \in \mathbb{Z}} \\ \\ \boldsymbol{x = 2\kappa\pi, \kappa \in \mathbb{Z}} \end{cases}$$

(γ) Έχουμε

$$2\eta\mu^2 x - 1 = 0 \Leftrightarrow 1 - 2\eta\mu^2 x = 0 \Leftrightarrow \sigma\upsilon\nu(2x) = 0 \Leftrightarrow \sigma\upsilon\nu(2x) = \sigma\upsilon\nu\left(\frac{\pi}{2}\right)$$
$$\Leftrightarrow \boldsymbol{2x = 2\kappa\pi \pm \frac{\pi}{2}, \kappa \in \mathbb{Z}} \Leftrightarrow \boldsymbol{x = \kappa\pi \pm \frac{\pi}{4}, \kappa \in \mathbb{Z}}$$

(δ) Έχουμε

$$2\varepsilon\varphi x + \tau\varepsilon\mu^2 x = 0 \Leftrightarrow \varepsilon\varphi^2 x + 2\varepsilon\varphi x + 1 = 0 \Leftrightarrow (\varepsilon\varphi x + 1)^2 = 0 \Leftrightarrow \varepsilon\varphi x = -1$$
$$\Leftrightarrow \varepsilon\varphi x = \varepsilon\varphi\left(-\frac{\pi}{4}\right) \Leftrightarrow \boldsymbol{x = \kappa\pi - \frac{\pi}{4}}, \qquad \boldsymbol{\kappa \in \mathbb{Z}}$$

(ε) Έχουμε

$$2\eta\mu^2 x + \eta\mu x - 1 = 0 \Leftrightarrow (\eta\mu x + 1)(2\eta\mu x - 1) = 0$$
$$\Leftrightarrow (\eta\mu x = -1) \vee \left(\eta\mu x = \frac{1}{2}\right)$$
$$\Leftrightarrow \left(\eta\mu x = \eta\mu\left(-\frac{\pi}{2}\right)\right) \vee \left(\eta\mu x = \eta\mu\left(\frac{\pi}{6}\right)\right)$$
$$\Leftrightarrow \left(\boldsymbol{x = 2\kappa\pi - \frac{\pi}{2}, \kappa \in \mathbb{Z}}\right) \vee \left(\boldsymbol{x = 2\kappa\pi + \frac{\pi}{6}, \kappa \in \mathbb{Z}}\right) \vee \left(\boldsymbol{x = 2\kappa\pi + \frac{5\pi}{6}, \kappa \in \mathbb{Z}}\right)$$

(στ) Έχουμε

$$\eta\mu x + \underbrace{\sigma\upsilon\nu(2x)}_{1-2\eta\mu^2 x} = 1 \Leftrightarrow \eta\mu x - 2\eta\mu^2 x = 0$$

$$\Leftrightarrow \eta\mu x(1 - 2\eta\mu x) = 0 \Leftrightarrow (\eta\mu x = 0) \vee \left(\eta\mu x = \frac{1}{2}\right)$$

$$\Leftrightarrow (\eta\mu x = \eta\mu 0) \vee \left(\eta\mu x = \eta\mu\left(\frac{\pi}{6}\right)\right)$$

$$\Leftrightarrow (x = 2\kappa\pi, \kappa \in \mathbb{Z}) \vee \left(x = 2\kappa\pi + \frac{\pi}{6}, \kappa \in \mathbb{Z}\right) \vee \left(x = 2\kappa\pi + \frac{5\pi}{6}, \kappa \in \mathbb{Z}\right)$$

$$\Leftrightarrow (\boldsymbol{x = \kappa\pi, \kappa \in \mathbb{Z}}) \vee \left(\boldsymbol{x = 2\kappa\pi + \frac{\pi}{6}, \kappa \in \mathbb{Z}}\right) \vee \left(\boldsymbol{x = 2\kappa\pi + \frac{5\pi}{6}, \kappa \in \mathbb{Z}}\right)$$

(ζ) Έχουμε

$$1 - \sigma\upsilon\nu x = \eta\mu x \cdot \eta\mu\left(\frac{x}{2}\right)$$
$$\Leftrightarrow 1 - \left(2\sigma\upsilon\nu^2\left(\frac{x}{2}\right) - 1\right) = 2\eta\mu\left(\frac{x}{2}\right) \cdot \sigma\upsilon\nu\left(\frac{x}{2}\right) \cdot \eta\mu\left(\frac{x}{2}\right)$$

$$\Leftrightarrow 2\left(1-\sigma\upsilon\nu^2\left(\frac{x}{2}\right)\right)=2\eta\mu^2\left(\frac{x}{2}\right)\cdot\sigma\upsilon\nu\left(\frac{x}{2}\right)$$
$$\Leftrightarrow 2\left(1-\sigma\upsilon\nu^2\left(\frac{x}{2}\right)\right)=2\left(1-\sigma\upsilon\nu^2\left(\frac{x}{2}\right)\right)\cdot\sigma\upsilon\nu\left(\frac{x}{2}\right)$$
$$\Leftrightarrow \left(1-\sigma\upsilon\nu^2\left(\frac{x}{2}\right)\right)\cdot\left(1-\sigma\upsilon\nu\left(\frac{x}{2}\right)\right)=0$$
$$\Leftrightarrow \left(1+\sigma\upsilon\nu\left(\frac{x}{2}\right)\right)\cdot\left(1-\sigma\upsilon\nu\left(\frac{x}{2}\right)\right)^2=0$$

και λύνοντας κατά τα γνωστά, βρίσκουμε ότι το σύνολο λύσεων είναι το $\{\boldsymbol{2\kappa\pi},\ \boldsymbol{\kappa}\in\mathbb{Z}\}$

(η) Έχουμε

$\eta\mu(10x)\sigma\upsilon\nu(6x)=\eta\mu(18x)\sigma\upsilon\nu(2x)$

$\Leftrightarrow \frac{1}{2}\big(\eta\mu(16x)+\eta\mu(4x)\big)=\frac{1}{2}\big(\eta\mu(20x)+\eta\mu(16x)\big)$

$\Leftrightarrow \eta\mu(16x)+\eta\mu(4x)=\eta\mu(20x)+\eta\mu(16x)$

$\Leftrightarrow \eta\mu(4x)=\eta\mu(20x)$

$\Leftrightarrow (4x=2\kappa\pi+20x,\kappa\in\mathbb{Z})\vee(4x=(2\kappa+1)\pi-20x,\kappa\in\mathbb{Z})$

$\Leftrightarrow (4x=2\kappa\pi+20x,\kappa\in\mathbb{Z})\vee(4x=(2\kappa+1)\pi-20x,\kappa\in\mathbb{Z})$

$\Leftrightarrow \left(\boldsymbol{x=-\frac{\kappa\pi}{8},\kappa\in\mathbb{Z}}\right)\vee\left(\boldsymbol{x=\frac{\kappa\pi}{12}+\frac{\pi}{24},\kappa\in\mathbb{Z}}\right)$

5.

(α) Λάθος. Η εξίσωση είναι αδύνατη

(β) Σωστό.

(γ) Λάθος. Μια λύση είναι π.χ. η $x=0$

(δ) Σωστό. Η εξίσωση γράφεται ισοδύναμα ως

$$\varepsilon\varphi^{2017}x-\varepsilon\varphi^{2016}x-2\varepsilon\varphi^2x-2\varepsilon\varphi x=0$$

Ακολούθως, αντικαταστήστε $x=\frac{\pi}{4}$ και χρησιμοποιήστε το ότι $\varepsilon\varphi\left(\frac{\pi}{4}\right)=1$.

6.

(α)

$$\sigma\upsilon\nu\left(3x+\frac{\pi}{4}\right)=\sigma\upsilon\nu\left(x-\frac{\pi}{3}\right)$$

$$\Leftrightarrow\left(3x+\frac{\pi}{4}=2\kappa\pi+x-\frac{\pi}{3},\kappa\in\mathbb{Z}\right)\vee\left(3x+\frac{\pi}{4}=2\kappa\pi-\left(x-\frac{\pi}{3}\right),\kappa\in\mathbb{Z}\right)$$

$$\Leftrightarrow\cdots\Leftrightarrow\left(\boldsymbol{x=\kappa\pi-\frac{7\pi}{24},\kappa\in\mathbb{Z}}\right)\vee\left(\boldsymbol{x=\frac{\kappa\pi}{2}+\frac{\pi}{48},\kappa\in\mathbb{Z}}\right)$$

Για $\kappa=0$, έχουμε $x=\frac{7\pi}{24}\notin(0,\pi)$ και $\boldsymbol{x=\frac{\pi}{48}\in(0,\pi)}$, για $\kappa=-1$, έχουμε $x=-\frac{31\pi}{24}\notin(0,\pi)$ και $\boldsymbol{x=\frac{25\pi}{48}\in(0,\pi)}$,

για $\kappa=1$, έχουμε

$$\boldsymbol{x=\frac{17\pi}{24}\in(0,\pi)}\ \text{και}\ \boldsymbol{x=\frac{25\pi}{48}\in(0,\pi)}$$

(β)

$$\sigma\varphi(4x-\pi)=\sigma\varphi\left(x-\frac{\pi}{3}\right)\Leftrightarrow\left(4x-\pi=\kappa\pi+x-\frac{\pi}{3},\kappa\in\mathbb{Z}\right)$$
$$\Leftrightarrow\left(x=\frac{1}{3}\left(\kappa\pi+\frac{2\pi}{3}\right),\kappa\in\mathbb{Z}\right)$$

Για $\kappa\in\mathbb{Z},\kappa>1$ οι τιμές είναι θετικές, άρα εκτός του $(-\pi,0)$.

Για $\kappa=-1$, έχουμε $\boldsymbol{x=-\frac{\pi}{9}\in(-\pi,0)}$

Για $\kappa=-2$, έχουμε $\boldsymbol{x=-\frac{4\pi}{9}\in(-\pi,0)}$

Για $\kappa=-3$, έχουμε $\boldsymbol{x=-\frac{7\pi}{9}\in(-\pi,0)}$

Για $\kappa=-4$, έχουμε $x=-\frac{10\pi}{9}\notin(-\pi,0)$

(γ)

$$\sigma\upsilon\nu\left(x+\frac{\pi}{3}\right)=\eta\mu x \Leftrightarrow \sigma\upsilon\nu\left(x+\frac{\pi}{3}\right)=\sigma\upsilon\nu\left(\frac{\pi}{2}-x\right)$$

$$\Leftrightarrow \left(x+\frac{\pi}{3}=2\kappa\pi+\frac{\pi}{2}-x,\kappa\in\mathbb{Z}\right)\vee\left(x+\frac{\pi}{3}=2\kappa\pi-\left(\frac{\pi}{2}-x\right),\kappa\in\mathbb{Z}\right)$$

$$\Leftrightarrow \left(2x=2\kappa\pi+\frac{\pi}{2}-\frac{\pi}{3},\kappa\in\mathbb{Z}\right)\Leftrightarrow \boldsymbol{x=\kappa\pi+\frac{\pi}{12}},\boldsymbol{\kappa}\in\mathbb{Z}$$

Για $\kappa=-1$, έχουμε $x=-\frac{11\pi}{12}<0$ Για $\kappa=0$, έχουμε $\boldsymbol{x=\frac{\pi}{12}}\in(\boldsymbol{0},\boldsymbol{2\pi})$

Για $\kappa=1$, έχουμε $\boldsymbol{x=\frac{13\pi}{12}}\in(\boldsymbol{0},\boldsymbol{2\pi})$ Για $\kappa=2$, έχουμε $x=\frac{25\pi}{12}>2\pi$