Δραστηριότητες σελ. 34 (Ενότητα 3.6.2: Τυποποίηση βασικών μορφών ολοκληρωμάτων)

1. (α) $$\int \sigma\upsilon\nu(7x)dx = \frac{1}{7}\int \sigma\upsilon\nu(7x)d(7x) = \frac{1}{7}\int \big(\eta\mu(7x)\big)'d(7x) = \frac{1}{7}\eta\mu(7x) + c$$

(β) $$\int \sigma\tau\varepsilon\mu^2(5x-3)dx = \frac{1}{5}\int \sigma\tau\varepsilon\mu^2(5x-3)d(5x-3) = -\frac{1}{5}\sigma\varphi(5x-3) + c$$

(γ) $$\int (6x-1)^{21}dx = \frac{1}{6}\int (6x-1)^{21}d(6x-1) = \frac{1}{6\times 22}(6x-1)^{22} + c = \frac{1}{132}(6x-1)^{22} + c$$

(δ) $$\int e^{4-9x}dx = -\frac{1}{9}\int e^{4-9x}d(4-9x) = -\frac{1}{9}e^{4-9x} + c$$

(ε) $$\int (e^{4x} - 2\cdot 4^{-3x})dx = \int e^{4x}dx - 2\cdot\int 4^{-3x}\,dx = \frac{1}{4}\int e^{4x}d(4x) + 2\cdot\int 4^{-3x}\,d(-3x)$$

$$= \frac{1}{4}e^{4x} + \frac{2\cdot 4^{-3x}}{3\cdot\ln 4} + c = \frac{1}{4}e^{4x} + \frac{2\cdot 4^{-3x}}{3\cdot\ln 2^2} + c = \frac{1}{4}e^{4x} + \frac{4^{-3x}}{3\cdot\ln 2} + c$$

(στ) $$\int \big(\eta\mu(4x) - \eta\mu(5x)\big)dx = \int \eta\mu(4x)dx - \int \eta\mu(5x)dx = -\frac{1}{4}\sigma\upsilon\nu(4x) + \frac{1}{5}\sigma\upsilon\nu(5x) + c$$

(ζ) $$\int \frac{1}{25x^2+1}dx = \int \frac{1}{(5x)^2+1}dx = \frac{1}{5}\int \frac{1}{(5x)^2+1}d(5x) = \frac{1}{5}\tau\circ\xi\varepsilon\varphi(5x) + c$$

(η) $$\int \frac{1}{\sqrt{4-x^2}}dx = \int \frac{1}{\sqrt{4\left(1-\frac{x^2}{4}\right)}}dx = \int \frac{1}{2\sqrt{1-\left(\frac{x}{2}\right)^2}}dx = \int \frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}}d\left(\frac{x}{2}\right) = \tau\circ\xi\eta\mu\left(\frac{x}{2}\right) + c$$

(θ) $$\int \frac{1}{(2x+1)^5}dx = \frac{1}{2}\int (2x+1)^{-5}d(2x+1) = -\frac{1}{2\times(5-1)(2x+1)^{5-1}} + c$$

$$= -\frac{1}{8(2x+1)^4} + c$$

(ι) $$\int x(x^2-3)^{17}dx = \int \left(\frac{1}{2}(x^2-3)\right)'(x^2-3)^{17}dx = \frac{1}{2}\int (x^2-3)^{17}d(x^2-3)$$

$$= \frac{1}{2\times 18}(x^2-3)^{18} + c = \frac{1}{36}(x^2-3)^{18} + c$$

(ια) $$\int \frac{\tau\circ\xi\varepsilon\varphi^3 x}{1+x^2}dx = \int \tau\circ\xi\varepsilon\varphi^3 x\cdot\frac{1}{1+x^2}dx = \int \tau\circ\xi\varepsilon\varphi^3 x\cdot(\tau\circ\xi\varepsilon\varphi x)'dx$$

$$= \int \tau\circ\xi\varepsilon\varphi^3 x d(\tau\circ\xi\varepsilon\varphi x) = \frac{1}{4}\tau\circ\xi\varepsilon\varphi^4 x + c$$

(ιβ) $$\int \frac{\ln^6 x}{x}dx = \int \ln^6 x\cdot\frac{1}{x}dx = \int \ln^6 x\cdot(\ln x)'dx = \frac{1}{7}\ln^7 x + c$$

(ιγ) $$\int \frac{2\eta\mu x}{\sqrt{1-\sigma\upsilon\nu x}}dx = 2\int \frac{\eta\mu x}{\sqrt{1-\sigma\upsilon\nu x}}dx = 2\int \frac{(1-\sigma\upsilon\nu x)'}{\sqrt{1-\sigma\upsilon\nu x}}dx = 2\int \frac{d(1-\sigma\upsilon\nu x)}{\sqrt{1-\sigma\upsilon\nu x}}$$

$$= 4\sqrt{1-\sigma\upsilon\nu x} + c$$

(ιδ) $$\int \frac{e^x}{(e^x-1)^4}dx = \int \frac{(e^x-1)'}{(e^x-1)^4}dx = \int \frac{d(e^x-1)'}{(e^x-1)^4} = -\frac{1}{3(e^x-1)^3} + c$$

(ιε) $$\int \frac{\tau\varepsilon\mu^2 x}{2+\varepsilon\varphi x}dx = \int \frac{(2+\varepsilon\varphi x)'}{2+\varepsilon\varphi x}dx = \int \frac{d(2+\varepsilon\varphi x)}{2+\varepsilon\varphi x} = \ln|2+\varepsilon\varphi x| + c$$

(ιστ) $\int \frac{3x}{x^2+3}dx = \frac{3}{2}\int \frac{2x}{x^2+3}dx = \frac{3}{2}\int \frac{(x^2+3)'}{x^2+3}dx = \frac{3}{2}\ln(x^2+3) + c$

(ιζ) $\int \sigma\varphi x\, dx = \int \frac{\sigma\upsilon\nu x}{\eta\mu x}dx = \int \frac{(\eta\mu x)'}{\eta\mu x}dx = ln|\eta\mu x| + c$

(ιη) $\int \frac{2+2\eta\mu x}{x-\sigma\upsilon\nu x}dx = 2\int \frac{1+\eta\mu x}{x-\sigma\upsilon\nu x}dx = 2\int \frac{(x-\sigma\upsilon\nu x)'}{x-\sigma\upsilon\nu x}dx = \ln|x-\sigma\upsilon\nu x| + c$

(ιθ) $\int \frac{6x+15}{x^2+5x-1}dx = 3\int \frac{2x+5}{x^2+5x-1}dx = 3\int \frac{(x^2+5x-1)'}{x^2+5x-1}dx = 3\ln|x^2+5x-1| + c$

(κ) $\int \sigma\tau\varepsilon\mu x\, dx = \int \sigma\tau\varepsilon\mu x \cdot \frac{\sigma\tau\varepsilon\mu x - \sigma\varphi x}{\sigma\tau\varepsilon\mu x - \sigma\varphi x}dx = \int \frac{\sigma\tau\varepsilon\mu^2 x - \sigma\tau\varepsilon\mu x \cdot \sigma\varphi x}{\sigma\tau\varepsilon\mu x - \sigma\varphi x}dx$

Αλλά, $(\sigma\tau\varepsilon\mu x - \varepsilon\varphi x)' = \sigma\tau\varepsilon\mu^2 x - \sigma\tau\varepsilon\mu x \cdot \sigma\varphi x$ και αρα

$$\int \sigma\tau\varepsilon\mu x\, dx = \ln|\sigma\tau\varepsilon\mu x - \sigma\varphi x| + c$$